

Speed addition and closed time cycle in Lorentz-non-invariant theories

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In theories, whose Lorentz invariance is violated by involvement of an external any-rank tensor, we show that the standard relativistic rule still holds true for summing the signal speed, understood as the group velocity of a wave, with the speed of the reference frame. Provided a superluminal signal is available, this observation enables one to arrange a closed time cycle and hence causality violation, notwithstanding the Lorentz noninvariance. Also an optical anisotropy of a moving medium, isotropic at rest, is revealed.

Keywords: Lorentz violation, causality, superluminal propagation speed, moving media, closed time circle

I. INTRODUCTION

A. About covariant description of Lorentz-non-invariant theories

In recent years much attention has been paid to the models, wherein the relativistic invariance does not take place. In such formulation this trend was initiated in [1]. It is aimed at detection of possible small deviations from the Standard Model, manifesting a violation of the Special Theory of Relativity. The object of investigation dealt with in those works is a special four-rank tensor peculiar to the vacuum, and not formed by the current fields. A special case of a Lorentz-non-invariant theory beyond the Standard Model is provided by space-time noncommutative theories (see the review articles [2, 3]). On the other hand, well within the Standard Model, we often consider theories where the invariance with respect to the Lorentz boosts, as well to the spatial rotations, is destroyed by the presence of external fixed tensors. These may be due to the presence of a medium or/and to external fields or/and to nontrivial metrics. In the first case the role of the external tensor, first-rank in this instance, is played by the four-velocity vector of the medium. In other cases these are the tensors of external fields or the ones characterizing the external metrics, also the noncommutativity tensor, when a non-commutative theory is under consideration.

In all these cases the theory may be given a Lorentz-covariant form. Namely, the action is formed as a Lorentz-scalar combination of fields and external tensors. So are the generating functionals of the Green functions and of the vertices (see, *e.g.*, [4]). The Green functions and the one-particle-irreducible vertices – second- and higher-rank polarization tensors – obtained by differentiation of the above functionals over the vector currents and fields, respectively, are formed as matrices constructed using the external tensors, apart from particle momenta or coordinates. In quantum electrodynamics with external electric and magnetic fields such program was realized in [5] (see also [6]). Perhaps, the first example of covariant representation of the photon polarization tensor in a (moving) medium making use of its four-velocity may be found in [7]. For the case of a medium in a magnetic field see [8], [6]. (A microscopic foundation of such a treatment of the four-velocity was given in [9] based on the temperature Green function formalism, see more comments in Subsection II C.) The covariant treatment of the noncommutativity matrix as a second-rank tensor was used with the same purpose in [10], where also the third-rank polarization tensor in external electric and magnetic fields responsible for the nonlinear processes of photon splitting and merging are built in the same manner. The covariant representation for the third-rank polarization tensor in a medium was used in [11].

The covariant approach is as a matter of fact a realization of the evident "extended relativity principle" to be read as: An inertial Lorentz reference frame A with an external tensor in it is equivalent to another reference frame B that moves with respect to A with a constant speed \mathbf{V} , provided that the external tensor in the frame B is the one, which has been Lorentz-transformed from that in A with the help of the speed \mathbf{V} .

The gauge invariant energy-momentum tensor in this approach is not symmetric, its antisymmetric part being responsible for nonconservation of the generators of angular momentum and Lorentz boosts [12].

In the present paper, following this approach we establish the transformation law of the group velocity of electromagnetic wave packet from one inertial frame to another to show that it is just the standard rule of relativistic summation of the two speeds: the group velocity of the packet and the relative speed of the two inertial reference frames.

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We are studying the group velocity, since this quantity is the speed of propagation of a wave packet [13], and it is significant for the issues of causality, discussed in Section III, although some authors often refer to the "phase" velocity, what is not grounded in this connection. The only reservation, usually made about the group velocity, is that it may exceed the speed of light near a resonance, causing the so-called abnormal dispersion. However, this reservation, too, can be lifted [14] by considering the real part of the complex group velocity as a function of real momentum, and not real group velocity plotted against the real part of complex momentum. As for the velocity of the wave front, referred to as the signal speed after Brillouin [15], it always propagates with the speed of light in the vacuum, since it is determined by the infinite-frequency limit $k_0 = \infty$, whereas in this limit no effect of Lorentz-violating factors, like an external field or a medium, can survive. Consequently, this speed is not important in discussing the causality issues.

The main derivation is given in Subsection II B, preceded by the description of the covariant approach in Subsection II A and followed by consideration of the interesting special case of light propagation in a moving medium, discussed as an illustration in Subsection II C. It is shown that the dielectric tensor of an isotropic medium is anisotropic in a moving frame, the principle axes being fixed by the directions of the wave-vector and the velocity \mathbf{V} . In the concluding Section III, among other issues of causality, we are discussing the arrangement of a closed time cycle in a Lorentz-non-invariant theory.

II. LORENTZ TRANSFORMATION OF THE GROUP VELOCITY

A. Polarization operator and dispersion equations

The (second pair of) Maxwell equations linearized above a homogeneous background for a free electromagnetic wave with the 4-vector potential $A^\rho(k)$ can be written in the momentum k_μ representation in the form

$$(k^2 g_{\mu\rho} - k_\mu k_\rho) A^\rho(k) - \Pi_{\mu\rho}(k) A^\rho(k) = 0, \quad (1)$$

where $\Pi_{\mu\rho}(k)$ is the polarization operator defined in the configuration space as

$$\Pi_{\mu\tau}(x, y) = \frac{\delta^2 \Gamma}{\delta A_\mu(x) \delta A_\tau(y)} \Big|_{F=\bar{F}=\text{const}}. \quad (2)$$

via the effective action Γ , the generating functional of one-particle-irreducible vertices, the polarization operator being one of them [4]. The effective action Γ here is admitted to depend, apart from external tensors, on any order space-time derivatives of the electromagnetic field tensor $F_{\mu\nu}$, but the electromagnetic fields are set constant $\bar{F}_{\mu\nu}$ (or zero, in the no external field case) after the differentiations. The Greek indices above span the Minkowski space.

Let us present the polarization operator in a diagonal form

$$\Pi_{\mu\tau}(k, p) = \delta(k - p) \Pi_{\mu\tau}(k), \quad \Pi_{\mu\tau}(k) = \sum_{a=1}^3 \kappa_a \frac{b_\mu^{(a)} b_\tau^{(a)}}{(b^{(a)})^2}, \quad (3)$$

where $b_\tau^{(a)}$ are its eigenvectors

$$\Pi_{\mu\tau} b_\tau^{(a)} = \kappa_a b_\mu^{(a)}, \quad a = 1, 2, 3, 4.$$

The effective action is a Lorentz scalar formed using all the external tensors and fields, and it does not depend on coordinates explicitly. Hence, the polarization operator (2) is a tensor in Minkowski space, while its eigenvalues κ_a are scalars. The appearance of the energy-momentum conserving delta-function in (3) is owing to the assumption that only space- and time-independent gauge-invariant Lorentz-violating agents are considered that do not violate the translation invariance. The fourth eigenvector is trivial, $b_\mu^{(4)} = k_\mu$, so the fourth eigenvalue vanishes, $\kappa_4 = 0$, as a consequence of the 4-transverseness of the polarization operator, $\Pi_{\mu\tau} k_\tau = 0$. All eigenvectors are mutually orthogonal, $b_\mu^{(a)} b_\mu^{(b)} \sim \delta_{ab}$, this means that the first three ones are 4-transversal, $b_\mu^{(a)} k_\mu = 0$.

The connection between $\Pi_{\mu\tau}$ and the four-rank tensor studied and attempted to be measured in the approach of Refs.[1] was discussed in [12].

The scalar eigenvalues κ_a may depend on all characteristic scalars in the theory, including k^2 and those which are formed by tensors of any rank peculiar to the medium or to the vacuum, when contracted with the photon 4-momentum k_μ , for instance $k \bar{F}^2 k$ or $k \theta \bar{F} k$ etc., where $\bar{F}_{\mu\nu}$ stands for external field strength tensor, and $\theta_{\mu\nu}$ for the non-commutativity tensor. Among these external tensors there may be the 4-velocity vector of a uniform medium, if

the latter is present. There are also momentum-independent scalars among arguments of \varkappa , like the external field invariants $\mathfrak{F} = -\frac{1}{4}\overline{F}^2$ and $\mathcal{G} = -\frac{1}{4}\overline{F}\widehat{F}$, where \widehat{F} is the dual electromagnetic tensor, but these are inessential for the present derivation. The photon dispersion laws for each of three polarization modes are to be found from the equations

$$k^2 = \varkappa_a(k), \quad a = 1, 2, 3, \quad (4)$$

which are the solvability conditions for Eq.(1). Equations (4) determine the frequency k_0 as a function of the wave-vector components k_i . The eigenvectors $b_\mu^{(a)}$ serve as 4-vector potentials for free eigenwaves. There are three eigen-modes, and not two, because possible massive vector particles are also included into the propagation equation (1). These may, for instance, be the electron-positron states (mutually free, or bound into the positronium atom), with which the photon unites into a mixed polariton state [16]. Another example of the third polarization degree of freedom, characteristic of massive vector particles, is supplied by the known longitudinal modes in a medium, see Subsection IIC for further comments.

The dielectric ε_{nj} tensor of the anisotropic "medium," to which the vacuum with the broken Lorentz invariance is equivalent irrespective of whether the real medium is present or not, is connected with the polarization tensor components as

$$\varepsilon_{nj} = \delta_{nj} + \frac{\Pi_{nj}}{k_0^2}, \quad n, j = 1, 2, 3. \quad (5)$$

The Lorentz-noninvariant vacuum manifests itself as an equivalent anisotropic "medium", since this tensor (5) is different from δ_{nj} . If a homogeneous medium, isotropic in its rest frame, is present alone in the Lorentz-invariant vacuum (i.e. where no external tensors besides the 4-velocity of the medium are included), it becomes anisotropic in a moving frame. (We shall elaborate this point in Subsection IIC below).

The refraction indices (in every eigenmode) are defined on solutions of dispersion equations (4) as

$$n_a(k_0, \mathbf{k}) = \frac{|\mathbf{k}|}{k_0} = \left(1 + \frac{\varkappa_a(k_0, \mathbf{k})}{k_0^2}\right)^{\frac{1}{2}}. \quad (6)$$

These are not Lorentz scalars, contrary to the eigenvalues \varkappa_a .

B. Derivation of the addition rule for speeds

Denote the momentum-dependent Lorentz invariants, $k^2 = -k_0^2 + k_i^2$ included, as I_s , $s = 1, 2, \dots$, their number depending on the problem. The derivative $\partial I_s / \partial k_\mu = P^{(s)\mu}$ is a Lorentz-vector. The group velocity of the photon is the 3-vector defined as the frequency differentiated over the wave vector

$$v_i^{\text{gr}} = \frac{\partial k_0}{\partial k_i}, \quad i = 1, 2, 3 \quad (7)$$

and calculated on a solution of the dispersion equation (4). We shall henceforward omit the indexing of the photon modes a in understanding that our derivation relates to any of the three. We find that (summation over all invariants I_s , whatever their number may be is meant)

$$v_i^{\text{gr}} = - \left(\frac{\partial(k^2 - \varkappa)}{\partial k_i} \right) \left(\frac{\partial(k^2 - \varkappa)}{\partial k_0} \right)^{-1} = \frac{2k_i - X_s P_i^{(s)}}{2k_0 - X_s P_0^{(s)}}, \quad (8)$$

where the quantities $X_s = \frac{\partial \varkappa}{\partial I_s}$ are Lorentz-invariant.

Let us now imagine that an inertial Lorentz frame W , to which the previous equations relate, moves with respect to the initial frame W' – to be marked with prime – with speed \mathbf{V} . We are going to demonstrate that the group velocity \mathbf{v}'^{gr} in the frame W' is connected with that in the frame W by the standard relativistic rule $\mathbf{v}' = \mathbf{v} \oplus \mathbf{V}$ of summing the speed of a signal \mathbf{v} with the speed of the reference frame, with the group velocity taken for \mathbf{v} . Namely, we shall demonstrate that

$$v_{\parallel}'^{\text{gr}} = v_{\parallel}^{\text{gr}} \oplus \mathbf{V} \equiv \frac{V + v_{\parallel}^{\text{gr}}}{1 + V v_{\parallel}^{\text{gr}}}, \quad (9)$$

$$\mathbf{v}'_{\perp}{}^{\text{gr}} = \mathbf{v}_{\perp}{}^{\text{gr}} \oplus \mathbf{V} \equiv \frac{\mathbf{v}_{\perp}{}^{\text{gr}} \sqrt{1-V^2}}{1 + V v_{\parallel}^{\text{gr}}}, \quad (10)$$

where the subscripts \parallel and \perp mark projections to the directions, parallel and orthogonal to \mathbf{V} .

We accept the view that the same physical process that underlies the signal propagating with the speed \mathbf{v} in the rest frame W should be responsible for its propagation with the speed $\mathbf{v} \oplus \mathbf{V}$ in the moving frame W' following equations of motion for the signal propagation covariantly transformed from W to W' . Namely, we adopt that the physical carrier of a signal is an electromagnetic wave process governed by equations (1), and that the role of the signal speed will be played by the group velocity of a propagating packet. We are essentially basing on the fact established in the previous Subsection, that the right-hand side \varkappa of the dispersion equation (4) is a Lorentz-invariant, to efficiently account for the mentioned Lorentz transformation of the electromagnetic signal speed.

Using eq.(8) in eq.(9) we get

$$\begin{aligned} v_{\parallel}^{\text{gr}} \oplus v &= \left(V + \frac{2k_{\parallel} - X_s P_{\parallel}^{(s)}}{2k_0 - X_s P_0^{(s)}} \right) \left(1 + V \frac{2k_{\parallel} - X_s P_{\parallel}^{(s)}}{2k_0 - X_s P_0^{(s)}} \right)^{-1} = \\ &= \frac{V(2k_0 - X_s P_0^{(s)}) + 2k_{\parallel} - X_s P_{\parallel}^{(s)}}{2k_0 - X_s P_0^{(s)} + V(2k_{\parallel} - X_s P_{\parallel}^{(s)})}. \end{aligned} \quad (11)$$

Bearing in mind that the Lorentz transformations for the vectors $\partial I_s / \partial k_{\mu} = P^{(s)\mu}$ (and analogous transformations for the momenta k^{μ}) between the two frames are

$$P'_{\parallel} = \frac{P_{\parallel} + V P_0}{\sqrt{1-V^2}}, \quad P'_0 = \frac{P_0 + V P_{\parallel}}{\sqrt{1-V^2}}, \quad (12)$$

we find from (11)

$$v_{\parallel}^{\text{gr}} \oplus \mathbf{V} = \frac{2k'_{\parallel} - X_s P'_{\parallel}{}^{(s)}}{2k'_0 - X_s P'_0{}^{(s)}}, \quad (13)$$

which is just $v'_{\parallel}{}^{\text{gr}}$, the (parallel projection of) the group velocity (8) calculated in the frame W' , taking into account that X_s are Lorentz-invariant. Thus, eq.(9) is proved.

Analogously, substituting eq.(8) into eq.(10) we obtain

$$\begin{aligned} \mathbf{v}_{\perp}{}^{\text{gr}} \oplus \mathbf{V} &= \left(\frac{2\mathbf{k}_{\perp} - X_s \mathbf{P}_{\perp}^{(s)}}{2k_0 - X_s P_0^{(s)}} \sqrt{1-V^2} \right) \left(1 + V \frac{2k_{\parallel} - X_s P_{\parallel}^{(s)}}{2k_0 - X_s P_0^{(s)}} \right)^{-1} = \\ &= \frac{(2\mathbf{k}_{\perp} - X_s \mathbf{P}_{\perp}^{(s)}) \sqrt{1-V^2}}{2k_0 - X_s P_0^{(s)} + V(2k_{\parallel} - X_s P_{\parallel}^{(s)})} = \frac{2\mathbf{k}'_{\perp} - X_s \mathbf{P}'_{\perp}{}^{(s)}}{2k'_0 - X_s P'_0{}^{(s)}}. \end{aligned} \quad (14)$$

We have used again the second equation in (12) for the transformation of $P_0^{(s)}$ and analogous transformation for k_0 , as well as the fact that vector components perpendicular to the relative speed \mathbf{V} of the two reference frames do not change under the Lorentz boost along \mathbf{V} . By comparing (14) with (8) we see that the former is just the perpendicular group velocity $v'_{\perp}{}^{\text{gr}}$ as calculated by the observer in the reference frame W' . Thus, the relativistic rule of summing speeds for the perpendicular component (10) has been proven, too.

The derivation of the relativistic law for speed summation above can by no means be repeated as applied to the phase velocity $v_{\text{ph}} = \frac{k_0}{|\mathbf{k}|}$, in correspondence with the well-known fact [18] that that velocity cannot serve as a signal speed and is not a 3-vector at all.

C. Moving isotropic medium

An important example of the above consideration is provided by an isotropic medium, whose presence violates the Lorentz invariance. The equations of electromagnetic field may be given a Lorentz-covariant form by introducing the

vector of four-velocity u_μ , $u^2 = -1$ of the medium, $u_0 = (1 - V^2)^{-\frac{1}{2}}$, $\mathbf{u} = \mathbf{V}(1 - V^2)^{-\frac{1}{2}}$, where \mathbf{V} is the velocity 3-vector of the moving medium. Then the scalars referred to in the previous Subsection are $I_1 = k^2$ and $I_2 = (uk)^2$.

The introduction of this vector gives the possibility to covariantly treat linear [7] and nonlinear [11] Maxwell equations of an initially isotropic medium also after it is set at motion with a constant speed as a whole and, moreover, placed in external field [8]. A microscopic theory justifying these receipts within the temperature Green function method in relativistic statistics due to [7] may be found in [9]. This theory is based on writing the density matrix in the form $\rho = \exp[-\beta(uP)]$, where β is the (Lorentz-scalar) inverse temperature, and P is the 4-momentum, in place of the standard expression $\rho = \exp[-\beta H]$, to which it reduces in the rest frame of the medium, where $u_\mu = (1, 0, 0, 0)$, $H = P^0$ is the Hamiltonian. Then the thermodynamical potential, playing the role of the Lagrangian density in plasma, is written as a Lorentz scalar with the use of the 4-velocity vector.

The extended relativity principle mentioned in Introduction now reads that any two inertial frames in a medium are equivalent after the speed of the medium is Lorentz-transformed from one frame to the other. In more sensual terms this means that an observer in a frame in motion relative to a medium, say, air, certainly realizes that he/she is moving, already because he/she may sense the wind. In this respect the usual relativity principle is violated together with the Lorentz-invariance. But after the motion of the medium with respect to the observer is excluded by its Lorentz transformation the situation for him/her returns to be equivalent to that viewed upon by an observer at rest with respect to the medium.

The most general covariant tensor representation [7] for the polarization operator of an isotropic homogeneous medium may be rewritten in the diagonal form [11], [17] as

$$\Pi_{\mu\nu}(k) = \kappa \sum_{b=1,2} \frac{c_\mu^{(b)} c_\nu^{(b)}}{(c^{(b)})^2} + \kappa_3 \frac{a_\mu a_\nu}{a^2}, \quad (15)$$

where

$$a_\mu = u_\mu k^2 - k_\mu(uk), \quad a^2 = k^2(k^2 - (uk)^2), \quad (au) = 0.$$

Vector $c_\mu^{(1)}$ is defined as an arbitrary 4-vector normal to the hyperplane, spanned by vectors k_μ and a_μ . Vector $c_\mu^{(2)} \equiv \varepsilon_{\mu\nu\rho\lambda} c_\nu^{(1)} a_\rho k_\lambda$ is also normal to that hyperplane and, also to $c_\mu^{(1)}$. The four vectors k_μ , a_μ , and $c_\mu^{(1,2)}$ are eigenvectors of the polarization operator

$$\begin{aligned} \Pi_\mu^\nu c_\nu^{(1,2)} &= \kappa_{1,2} c_\mu^{(1,2)}, \quad \kappa_1 = \kappa_2 = \kappa \\ \Pi_\mu^\nu a_\nu &= \kappa_3 a_\mu, \quad \Pi_\mu^\nu k_\nu = 0. \end{aligned} \quad (16)$$

Only three of them are involved in the decomposition (15), since one eigenvalue is zero, in accord with the transversality $\Pi_\mu^\nu k_\nu = 0$.

The three basis vectors a_μ , and $c_\mu^{(1,2)}$ are 4-vector potentials of the electromagnetic eigen-waves. The orientations of the corresponding electric, $e_i \sim k_0 a_i - k_i a_0$ (or $e_i \sim k_0 c_i^{(1,2)} - k_i c_0^{(1,2)}$) and magnetic ($h_i = \epsilon_{ijn} k_j a_n$ or $h_i = \epsilon_{ijn} k_j c_n^{(1,2)}$) fields, calculated basing on these vector-potentials, are described in detail in [11]. In the Lorentz frame, where the medium is at rest, modes 1 and 2 are electromagnetic waves transversely-polarized in the plane orthogonal to the wave vector \mathbf{k} , while mode 3 is purely electric wave, its magnetic field being equal to zero and electric field being longitudinally polarized along \mathbf{k} , $\mathbf{e} \sim \mathbf{k}(k_0^2 - \mathbf{k}^2)$. This wave may be realized provided the dispersion equation has a massive solution with $(k_0^2 - \mathbf{k}^2) \neq 0$. We are saying this with the only purpose to illustrate, how these two well-known facts appear in our formalism.

The degeneracy $\kappa_1 = \kappa_2$ in (16) reflects the symmetry of the problem under rotations around the direction of the speed \mathbf{V} .

The dispersion equations (4) take the form

$$k^2 = \kappa_{1,2}(k^2, (uk)^2), \quad k^2 = \kappa_3(k^2, (uk)^2),$$

wherein we explicitly indicated the dependence of the eigenvalues on two Lorentz-scalars k^2 and $(uk)^2$.

In accord with the representation (15), the space-space part of the polarization tensor, Π_{nj} , apart from the unit tensor δ_{nj} , is formed by the wave vector \mathbf{k} and the velocity of the moving medium \mathbf{V} :

$$\Pi_{nj} = A\delta_{nj} + Bk_n k_j + C(k_n V_j + V_n k_j) + D V_n V_j, \quad (17)$$

where the coefficients A , B , C and D are certain rotational scalars, functions of \mathbf{k}^2 , V , $(\mathbf{V} \cdot \mathbf{k})$ and k_0 . Therefore, the vector \mathbf{d} orthogonal to the plane spanned by these two vectors, $(\mathbf{d} \cdot \mathbf{k}) = (\mathbf{d} \cdot \mathbf{V}) = 0$, is a universal eigenvector of the

dielectric tensor (5)

$$\varepsilon_{nj}d_j = \frac{A}{k_0^2}d_i.$$

The orientations of the other two eigenvalues in the above plane depend on individual orientations of the vectors \mathbf{k} and \mathbf{V} . This situation is typical for crystals of monoclinic system [18].

We conclude this Subsection with a statement that an isotropic medium behaves itself as anisotropic, when it moves. Its dielectric tensor has three different eigenvalues, all depending on the speed of the medium, and three optical axes determined by direction of that speed. The refraction indices depend on the reference frame, too. There are three eigenwaves with different dispersion laws. Transparent isotropic bodies moving with relativistic speed are birefringent.

Eqs.(9), (10) read that, in a moving frame, the speed of every eigenwave, considered as its group velocity, is obtained as a relativistic sum of its group velocity in the rest frame with the speed of the medium. The same statement concerning a medium, anisotropic already in the reference frame, where it is at rest, cannot be derived from equations of the previous Subsection, since it is unclear if 3-dimensional tensors, responsible for such anisotropy, can be given a relativistic-covariant extension with the help of the vector u_μ .

III. CONCLUDING REMARKS

We have demonstrated that when an electromagnetic wave packet is the physical carrier of information, its group velocity obeys the standard relativistic law (9), (10) for speed summation irrespective of whether the Lorentz invariance of the vacuum holds true or is violated by the presence of any external vector(s) or any-rank tensor(s). Thereby, we have shown that the light propagation process that is governed by electromagnetic field equations depending on that (those) tensor(s), guarantees that the group velocity in the rest frame and in the moving frame are related according to the relativistic law of signal's speed transformation. This indicates that, in the Lorentz-violated vacuum, the same as in the Lorentz-symmetrical case [13], the group velocity possesses the important property of a signal speed. In particular, when the group velocity does not exceed $c = 1$ (the speed of light in the Lorentz-invariant vacuum), and the speed of the reference frame does not exceed unity either, the resulting group velocity in the moving frame remains lesser than unity. So, normally, the causality survives the relativistic invariance violation.

However, it sometimes happens that a superluminal signal appears resulting from dynamical calculations in theories with violated Lorentz-invariance. The examples are given by light propagation in external metrics [19] and noncommutative electrodynamics [20], [10], also in QED with an extraordinary strong external field [21] and in other systems. (Although in [20] the phase velocity was used as a criterion for superluminality, the conclusion of the authors turns out to be correct, since it relates, as matter of fact, to the group velocity, as well). There is a temptation to dismiss the grave character of such results referring to the idea that as long as the relativistic invariance has been abandoned there is no reason to concern about causality any longer following the principle expressed in the Russian proverb: "Sniavshi golovu, po volosam ne plachut!" or in its German rationalized equivalent: "Ist der Kopf abgeschlagen, wird niemand nach dem Hute fragen?" This means approximately that the one who is beheaded should not mourn over his hair or hat. This stand is grounded by the statement [22] that thanks to the "extended relativity principle" as we were describing it in Subsection II C one cannot arrange the closed time cycle in a Lorentz-noninvariant theory even when a superluminal signal is at one's disposal. Intuitively, this statement seems paradoxical, because it admits that, for instance, faster-than-light propagation of acoustic signal in an infinitely extended perfectly rigid body, whose presence certainly singles out its reference frame among other frames and hence violates the relativistic invariance, may be compatible with causality.

We keep, however, to the opposite point of view expressed in [23]; we stress that the causality issues remain meaningful beyond the Lorentz-invariance, too. Our results allow to state that when the superluminal signal is carried by the electromagnetic wave packet in a Lorentz-noninvariant theory, the causality principle is destroyed in the same way as in a Lorentz-invariant theory. The point is that the "time machine", i.e. the closed time cycle, can be constructed in a gedanken experiment notwithstanding the lack of relativistic invariance. To be more specific, let a signal be emitted by an emitter at rest in the reference frame, designed as a primed frame in Subsection II B. Let it be emitted in the origin $x' = 0$ at the time moment $t' = 0$ and then propagate with the superluminal speed $v' > 1$ along the axis 1 to come to a certain point (x'_1, t') with $x'_1 = v't' > t'$. Since this point is space-like, the Lorentz boost exists $x_1 = (x'_1 + Vt')/(1 - V^2)^{1/2}$, $t = (t' + Vx'_1)/(1 - V^2)^{1/2}$ to an (unprimed) frame moving with the negative underluminal speed $1 > |V| > 1/v'$ that reverses the sign of the time $t < 0$. To let the signal reflected by a detector at rest in the unprimed frame come back to the origin and close the time cycle thereby, the reflected signal should have the speed $|v| = x/t = (x'_1 + Vt')/(t' + Vx'_1)$, which is just our equation (9) for the group velocity in the moving frame. Now we may perform the inverse boost to come back to the frame equivalent to the initial primed frame. The origin is not transformed by this boost. So, once the signal transmission is realized via a wave process governed by the

Lorentz-invariant (with external tensors included) dispersion law (4), the paradoxical influence of a consequence on its cause is achieved despite the lack of Lorentz invariance. Consequently, situations where the group velocity turns out to be greater than unity should be recognized as contradicting the causality principle in a theory with the Lorentz symmetry violated by an external tensor (as well as in a Lorentz-invariant theory, of course).

However, it was pointed in Ref. [10], where the excess of the group velocity of an electromagnetic wave in noncommutative electrodynamics with external constant fields over unity was analyzed, that its extreme smallness, $v^{\text{gr}} - 1 \ll 1$, results in the fact that the speed of the emitter and detector necessary to realize the paradox of the *influence on the past* becomes too large, beyond any existing human experience. In other words, for realizing the closed circle it is not enough to have a superluminal signal at one's disposal, but also some macroscopical bodies must move with the speed, smaller than, but very close to that of light, $1 > |V| > 1/v$. The present-day knowledge does not cover the range of such speeds of macroscopic instruments. Therefore, deviation from the causality principle, logically absurd as it is, should not be completely ruled out on the basis of existing experience, provided there is but a tiny excess of the signal speed over that of light.

Now we extend that analysis to the superluminal electromagnetic wave solutions in black hole metrics, discovered within the low-momentum one-electron-positron-loop approximation in [19]. For the Schwarzschild metrics the speed excess makes $\Delta v \sim \frac{\alpha}{30\pi} \frac{R_{\text{gr}}}{r} \left(\frac{\lambda_C}{r}\right)^2$, where $\alpha = \frac{1}{137}$ is the fine structure constant, R_{gr} is the gravitational radius of the black hole, $\lambda_C \sim 2.4 \times 10^{-10} \text{cm}$ is the electron Compton wave length, and r is the distance from the center of the star. When the latter is equal to its gravitational radius, $r = R_{\text{gr}}$, one gets $\Delta v \sim \frac{\alpha}{30\pi} \left(\frac{\lambda_C}{R_{\text{gr}}}\right)^2$. Even for the smallest black holes with their masses of the order of a few solar masses, where $R_{\text{gr}} \sim 5 \times 10^5 \text{cm}$, this excess makes $\Delta v \sim 2 \times 10^{-35}$, which is much smaller than the admissible estimates resulting from the noncommutative theory in [10] would admit. This means that a tachyon, if emitted from inside of a black hole, can overcome its horizon with a very small excess of its speed over the unity to be reduced to nothing as it goes away from it. On the contrary, deep inside the black hole, where r is small, the speed excess may be large, only limited by what the approximations in [19] allow, i.e. $\Delta v \ll 1$. One can imagine that a tachyon emitted from the depth of the black hole meets, as it moves toward the horizon, the matter falling inside with the speed approaching that of light. The latter may reflect it back towards its source to form the necessary "detail" of the time machine depicted above. Thus a prospect of developing an acausal electrodynamics of black holes arises based not only on retarded, but also on advanced interaction.

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